SUPPLEMENTARY MATERIALS: STABILITY AND CONTINUITY IN ROBUST OPTIMIZATION*

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Appendix A. Summary of Notation.

TABLE SM1
Summary of notation

Notation	Name	Definition
T	index of LSIO problem	
$\boldsymbol{\pi}$	LSIO problem	$\boldsymbol{\pi} := (\mathbf{c}, (\mathbf{a}_t, b_t)_{t\in T}) = (\mathbf{c}, (a, b))$
$\boldsymbol{\sigma}$	constraint system	$\boldsymbol{\sigma} := (\mathbf{a}_t, b_t)_{t\in T} = (a, b)$
Π	Set of LSIO problems	$\Pi := \mathbb{R}^n \times (\mathbb{R}^n \times \mathbb{R})^T$
Σ	Set of constraint systems	$\Sigma := (\mathbb{R}^n \times \mathbb{R})^T$
$F(\boldsymbol{\sigma})$	feasible solution set of $\boldsymbol{\pi}$	$F(\boldsymbol{\sigma}) := \{x \in \mathbb{R}^n : \langle \mathbf{a}_t, \mathbf{x}\rangle \geq b_t,\ \forall t \in T\}$
$\nu(\boldsymbol{\pi})$	optimal value of $\boldsymbol{\pi}$	$\nu(\boldsymbol{\pi}) := \inf_{x\in\mathbb{R}^n}\{\langle \mathbf{c}, \mathbf{x}\rangle : \mathbf{x} \in F(\boldsymbol{\sigma})\}$
$F^{\text{opt}}(\boldsymbol{\pi})$	optimal solution set of $\boldsymbol{\pi}$	$F^{\text{opt}}(\boldsymbol{\pi}) := \{x \in \mathbb{R}^n : \langle \mathbf{c}, \mathbf{x}\rangle = \nu(\boldsymbol{\pi})\}$
$F^{\epsilon\text{-opt}}(\boldsymbol{\pi})$	ϵ-optimal solution set of $\boldsymbol{\pi}$	$F^{\epsilon\text{-opt}}(\boldsymbol{\pi}) := \{\mathbf{x} \in F(\boldsymbol{\sigma}) : \langle \mathbf{c}, \mathbf{x}\rangle \leq \nu(\boldsymbol{\pi}) + \epsilon\}$
$d(\mathbf{x}, C)$	Point-set distance	$\inf_{\mathbf{y}\in C} d(\mathbf{x}, \mathbf{y})$
$d_H(C, D)$	Hausdorff distance	$\max\left\{\sup_{\mathbf{u}\in U}\inf_{\mathbf{v}\in V}\Vert\mathbf{u}-\mathbf{v}\Vert, \sup_{\mathbf{v}\in V}\inf_{\mathbf{u}\in U}\Vert\mathbf{u}-\mathbf{v}\Vert\right\}.$
$\mathbf{d}_r(C, D)$		$\max_{\Vert\mathbf{x}\Vert\leq r} \vert d(\mathbf{x}, C) - d(\mathbf{x}, D)\vert$
$\widehat{\mathbf{d}}_r(C, D)$	(see Definition 7.1)	$\inf_{\eta\geq 0}\left\{\eta \,\middle\vert\, \begin{array}{l} C\cap rB \subseteq D + \eta B \\ D \cap rB \subseteq C + \eta B\end{array}\right\}.$
$\mathbf{d}_\natural(\widehat{\mathbf{U}}, \widehat{\mathbf{V}})$		$\sup_{\alpha\in I} d_H(U_\alpha, V_\alpha)$
$e(C, D)$	Excess of C on D	$\sup_{\mathbf{x}\in C}\inf_{\mathbf{y}\in D} d(\mathbf{x}, \mathbf{y})$
$(C)_r$		$C \cap rB$
$\boldsymbol{\sigma}_1 \sim_\Sigma \boldsymbol{\sigma}_2$	$\boldsymbol{\sigma}_1$ is Σ-equivalent to $\boldsymbol{\sigma}_2$	$\{(\mathbf{a}_t^1, b_t^1), t \in T\} = \{(\mathbf{a}_t^2, b_t^2), t \in T\}$
$\boldsymbol{\pi}_1 \sim_\Pi \boldsymbol{\pi}_2$	$\boldsymbol{\pi}_1$ is Π-equivalent to $\boldsymbol{\pi}_2$	$\boldsymbol{\sigma}_1 \sim_\Sigma \boldsymbol{\sigma}_2$ and $c^1 = c^2$
Σ_f	set of feasible $\boldsymbol{\sigma}$	$\Sigma_f := \{\boldsymbol{\sigma} \in \Sigma : F(\boldsymbol{\sigma}) \neq \varnothing\}$
Σ_i	set of infeasible $\boldsymbol{\sigma}$	$\Sigma_i := \Sigma\backslash\Sigma_f$
Σ_∞		$\Sigma_\infty := \{\boldsymbol{\sigma} \in \Sigma : \delta^\Sigma(\boldsymbol{\sigma}, \text{bd}(\Sigma_f)) = +\infty\}$
Π_f	set of feasible $\boldsymbol{\pi}$	$\Pi_f := \{\boldsymbol{\pi} = (\mathbf{c}, \boldsymbol{\sigma}) \in \Pi : \boldsymbol{\sigma} \in \Sigma_f\}$
Π_i	set of infeasible $\boldsymbol{\pi}$	$\Pi_i := \Pi\backslash\Pi_f$
Π_∞		$\Pi_\infty := \{\boldsymbol{\pi} \in \Pi : \delta^\Pi(\boldsymbol{\pi}, \text{bd}(\Pi_f)) = +\infty\}$
Π_s	set of solvable $\boldsymbol{\pi}$	$\Pi_s := \{\boldsymbol{\pi} = (\mathbf{c}, \boldsymbol{\sigma}) \in \Pi : F^{\text{opt}}(\boldsymbol{\pi}) \neq \varnothing\}$
$\delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)$	distance b/w $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$	$\delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2) := \sup_{t\in T}\left\Vert \begin{pmatrix} \mathbf{a}^1(t) \\ b^1(t)\end{pmatrix} - \begin{pmatrix} a^2(t) \\ b^2(t)\end{pmatrix}\right\Vert$
$\delta^\Pi(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2)$	distance b/w $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$	$\delta^\Pi(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2) := \max\{\Vert\mathbf{c}^1 - \mathbf{c}^2\Vert, \delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)\}$

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TABLE SM2
Summary of notation - continued

Notation	Name	Definition
$A(\boldsymbol{\pi})$		$A(\boldsymbol{\pi}) := \text{conv}(\{\mathbf{a}_t, t \in T\})$
$R(\boldsymbol{\pi})$		$R(\boldsymbol{\pi}) := \{-b_t, t \in T; \nu(\boldsymbol{\pi})\}$
$Z^-(\boldsymbol{\pi})$		$Z^-(\boldsymbol{\pi}) := \text{conv}(\{\mathbf{a}_t, t \in T; -c\})$
$H(\boldsymbol{\sigma})$		$H(\boldsymbol{\sigma}) := \text{conv}\left(\left\{\begin{pmatrix} \mathbf{a}_t \\ b_t \end{pmatrix}\right\}_{t\in T}\right) + \left\{\begin{pmatrix} 0_n \\ -\mu \end{pmatrix}\right\}_{\mu \geq 0}$
$\varphi(\lambda)$		$\varphi(\lambda) = \varphi_*(\lambda) := \sqrt{1+\lambda^2}$
$\psi(\alpha)$		$(1+\alpha)\sqrt{1+\alpha^2}$
$\widehat{\rho}(\boldsymbol{\pi}_0)$		$\frac{\sup R(\boldsymbol{\pi}_0)}{d_*(0_n, \text{bd}(Z^-(\boldsymbol{\pi}_0)))}$
$\beta(\boldsymbol{\pi}_0, \epsilon)$		$\frac{\psi(\widehat{\rho}(\boldsymbol{\pi}_0))}{\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) - \epsilon}$
$\gamma(\boldsymbol{\pi}_0, \epsilon)$		$\varphi_*(0)(\widehat{\rho}(\boldsymbol{\pi}_0) + \beta(\boldsymbol{\pi}_0)\epsilon) + \\|\mathbf{c}^0\\|_* \beta(\boldsymbol{\pi}_0, \epsilon)$
$\mu(\boldsymbol{\pi}_0, \epsilon)$		$\varphi(0)\frac{\sup R(\boldsymbol{\pi}_0) + \epsilon \max\{1, \gamma(\boldsymbol{\pi}_0, \epsilon)\}}{d(0_n, \text{bd}(Z^-(\boldsymbol{\pi}_0))) - \epsilon}$
$L(\boldsymbol{\pi}_0, \epsilon)$	Lipschitz constant	$\varphi_*(0)\left((\epsilon + \\|\mathbf{c}^0\\|)\frac{\psi(\mu(\boldsymbol{\pi}_0, \epsilon))}{\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) - \epsilon} + \mu(\boldsymbol{\pi}_0, \epsilon)\right)$
$\delta_r(f_1, f_2)$		$\mathbf{d}_r(\text{epi} f_1, \text{epi} f_2)$
$\widehat{\delta}_r(f_1, f_2)$		$\widehat{\mathbf{d}}_r(\text{epi} f_1, \text{epi} f_2)$
$\widehat{\delta}_r^+(f_1, f_2)$		$\inf_{\eta \geq 0}\left\{\eta \,\middle\vert\, \begin{array}{l} \min_{B(x,\eta)} f_2 \leq \max\{f_1(\mathbf{x}), -r\} + \eta \\ \min_{B(x,\eta)} f_1 \leq \max\{f_2(\mathbf{x}) - r\} + \eta \end{array} \forall \mathbf{x} \in rB\right\}$

Appendix B. Facts about sets of LSIO problems.

LEMMA B.1. *The sets* Σ_f, $\text{bd}(\Sigma_f)$ *and* Σ_i *are non-empty. Furthermore, the sets* Π_f, $\text{bd}(\Pi_f)$, Π_i, *and* Π_s *are non-empty.*

Proof. Consider constraint system $\boldsymbol{\sigma}_0$ with $(0_n, 0)$, that is, $\langle 0_n, \mathbf{x}\rangle \geq 0$, as the only constraint (up to some multiplicity). Note that $\boldsymbol{\sigma}_0$ is in both Σ_f and $\text{bd}(\Sigma_f)$; the latter inclusion is obtained by recognizing that the constraint systems $\boldsymbol{\sigma}_\epsilon$ with $\langle 0_n, \mathbf{x}\rangle \geq \epsilon$ as the only constraint satisfy $\boldsymbol{\sigma}_\epsilon \in \Sigma_i$ for any $\epsilon > 0$. The fact that $\boldsymbol{\sigma}_\epsilon \in \Sigma_i$ for any $\epsilon > 0$ also shows that Σ_i is non-empty. The same examples with the cost function $c = 0_n$ for all of them prove that Π_f, $\text{bd}(\Pi_f)$, Π_i, and Π_s are non-empty. ☐

LEMMA B.2. *The following hold:*

$$\boldsymbol{\sigma} \in \text{cl}(\Sigma_f) \implies \boldsymbol{\sigma} \notin \Sigma_\infty$$
$$\boldsymbol{\pi} \in \text{cl}(\Pi_f) \implies \boldsymbol{\pi} \notin \Pi_\infty.$$

Proof. $\underline{\boldsymbol{\sigma} \in \text{cl}(\Sigma_f) \implies \boldsymbol{\sigma} \notin \Sigma_\infty}$. By Corollary 1 from [6], if $\boldsymbol{\sigma} \in \Sigma_f$ then $\delta^\Sigma(\boldsymbol{\sigma}, \Sigma_i) = \delta^\Sigma(\boldsymbol{\sigma}, \text{bd}(\Sigma_f))$. Remark 2 from [6] states

$$+\infty > \delta^\Sigma(\boldsymbol{\sigma}, \Sigma_i) = \delta^\Sigma(\boldsymbol{\sigma}, \text{bd}(\Sigma_f)).$$

Thus, by definition, $\boldsymbol{\sigma} \notin \Sigma_\infty$. If $\boldsymbol{\sigma} \in \text{bd}(\Sigma_f)$, then by definition $\boldsymbol{\sigma} \notin \Sigma_\infty$.

$\underline{\boldsymbol{\pi} \in \text{cl}(\Pi_f) \implies \boldsymbol{\pi} \notin \Pi_\infty}$. If $\boldsymbol{\pi} := (\mathbf{c}, \boldsymbol{\sigma}) \in \Pi_f$, by definition, $\boldsymbol{\sigma} \in \Sigma_f$. By the first statement, $\boldsymbol{\sigma} \notin \Sigma_\infty$. By definition,

$$\delta^\Pi(\boldsymbol{\pi}, \text{bd}(\Pi_f)) := \inf_{\boldsymbol{\pi}' \in \text{bd}(\Pi_f)} \delta^\Pi(\boldsymbol{\pi}, \boldsymbol{\pi}') = \inf_{\boldsymbol{\pi}' := (\mathbf{c}', \boldsymbol{\sigma}') \in \text{bd}(\Pi_f)} \max\{\|\mathbf{c} - \mathbf{c}'\|, \delta^\Sigma(\boldsymbol{\sigma}, \boldsymbol{\sigma}')\}.$$

Consider the restricted subset of points in $\text{bd}(\Pi_f)$ such that $\boldsymbol{\pi}' = (\mathbf{c}, \boldsymbol{\sigma}')$, i.e. the problems with all the *same* cost function as $\boldsymbol{\pi}$, which we denote $\text{bd}(\Pi_f) \cap \{\boldsymbol{\pi}' : \mathbf{c}' = \mathbf{c}\}$.

Then:

$$\delta^{\Pi}(\boldsymbol{\pi}, \mathrm{bd}(\Pi_f)) = \inf_{\boldsymbol{\pi}' := (\mathbf{c}', \boldsymbol{\sigma}') \in \mathrm{bd}(\Pi_f)} \max\{\|\mathbf{c} - \mathbf{c}'\|, \delta^{\Sigma}(\boldsymbol{\sigma}, \boldsymbol{\sigma}')\}, \tag{B.1}$$

$$\leq \inf_{\boldsymbol{\pi}' \in \mathrm{bd}(\Pi_f) \cap \{\boldsymbol{\pi}' \,:\, \mathbf{c}' = \mathbf{c}\}} \max\{0, \delta^{\Sigma}(\boldsymbol{\sigma}, \boldsymbol{\sigma}')\}. \tag{B.2}$$

The inequality (B.2) is the same as $\delta^{\Sigma}(\boldsymbol{\sigma}, \mathrm{bd}(\Sigma_f))$. Now $\delta^{\Sigma}(\boldsymbol{\sigma}, \mathrm{bd}(\Sigma_f)) < +\infty$ implies that $\delta^{\Pi}(\boldsymbol{\pi}, \mathrm{bd}(\Pi_f)) < +\infty$. Thus, by definition, $\boldsymbol{\pi} \notin \Pi_\infty$. If $\boldsymbol{\pi} \in \mathrm{bd}(\Pi_f)$, then by definition $\boldsymbol{\pi} \notin \Pi_\infty$. ∎

Appendix C. Proofs for Section 4.

C.1. Proof of Theorem 4.4.

Proof. We first prove that $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ is a well-defined LSIO problem and is equivalent to $\mathbf{RO}(\widehat{\mathbf{U}})$, omitting the analogous proof for $\boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}$ and $\mathbf{RO}(\widehat{\mathbf{V}})$. Then, we prove that $\delta^{\Pi}(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}) = \mathbf{d}_\natural(\widehat{\mathbf{U}}, \widehat{\mathbf{V}})$.

To show that $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ is well-defined, we must show that $\boldsymbol{\sigma}_{\widehat{\mathbf{U}},\widehat{\mathbf{V}}}$ maps *every* element in T to *exactly one* constraint (i.e., vector in $\mathbb{R}^{n+1}$). For any given α, when $(t, s) \in U_\alpha$ or $(t, s) \notin U_\alpha \cup V_\alpha$, it is clear that (t, s) maps to exactly one constraint, either (t, s) or $(0_n, -\rho)$. When $(t, s) \in V_\alpha \backslash U_\alpha$, the existence and uniqueness of $\mathrm{argmin}_{(\mathbf{u}_a, u_b) \in U_\alpha} d((\mathbf{u}_a, u_b), (t, s))$ follows since U_α is compact and convex and $d(\cdot, \cdot)$ is the Euclidean norm.

To show that $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ is equivalent to $\mathbf{RO}(\widehat{\mathbf{U}})$, it suffices to show that $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ has exactly the same feasible solution set and cost function as $\mathbf{RO}(\widehat{\mathbf{U}})$. Let $\alpha \in I$ be chosen. First, if $(t, s) \in U_\alpha$, then $\boldsymbol{\sigma}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}((\alpha, t, s)) := (t, s)$, thus, since we chose arbitrary α, every constraint in $\mathbf{RO}(\widehat{\mathbf{U}})$ is listed in $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$. Second, since $\mathrm{argmin}_{(\mathbf{u}_a, u_b) \in U_\alpha} d((\mathbf{u}_a, u_b), (t, s)) \in U_\alpha$ if $(t, s) \in V_\alpha \backslash U_\alpha$, it follows that

$$\boldsymbol{\sigma}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}((\alpha, t, s)) := \mathrm{argmin}_{(\mathbf{u}_a, u_b) \in U_\alpha} d((\mathbf{u}_a, u_b), (t, s))$$

is a redundant constraint. Lastly, if $(t, s) \notin U_\alpha \cup V_\alpha$, then $\boldsymbol{\sigma}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}((\alpha, t, s)) := (0_n, -\rho)$, which is a trivial constraint. Thus, every constraint in $\mathbf{RO}(\widehat{\mathbf{U}})$ is included in $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ and all the other constraints of $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ do not constrain the feasible solution set any further. Thus, the feasible solution set of $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ is the same as $\mathbf{RO}(\widehat{\mathbf{U}})$, and since the cost function is the same by construction, it follows that the optimal value and optimal solution set of $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ and $\mathbf{RO}(\widehat{\mathbf{U}})$ are also the same.

Lastly, we show that $\delta^{\Pi}(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}) = d_H(\widehat{\mathbf{U}}, \widehat{\mathbf{V}})$. Observe that for every $(\alpha, t, s) \in T$:

$$\|\boldsymbol{\sigma}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}((\alpha, t, s)) - \boldsymbol{\sigma}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}((\alpha, t, s))\| = \tag{C.1}$$
$$\begin{cases} 0 & \text{if } (t, s) \in U_\alpha \cap V_\alpha \\ \min_{(\mathbf{v}_a, v_b) \in V_\alpha} d((\mathbf{v}_a, v_b), (t, s)) & \text{if } (t, s) \in U_\alpha \backslash V_\alpha \\ \min_{(\mathbf{u}_a, u_b) \in U_\alpha} d((\mathbf{u}_a, u_b), (t, s)) & \text{if } (t, s) \in V_\alpha \backslash U_\alpha \\ 0 & \text{if } (t, s) \notin U_\alpha \cup V_\alpha, \end{cases}$$

where the minimum in the case $t \in U_\alpha \backslash V_\alpha$ follows because

$$d((t, s), \mathrm{argmin}_{(\mathbf{v}_a, v_b) \in V_\alpha} d((\mathbf{v}_a, v_b), (t, s))) = \min_{(\mathbf{v}_a, v_b) \in V_\alpha} d((\mathbf{v}_a, v_b), (t, s)),$$

by definition, and analogously for the case $t \in V_\alpha \backslash U_\alpha$. Taking the supremum of (C.1) over $\mathbf{t} = (\alpha, t, s) \in T$:

$$\sup_{\mathbf{t}\in T} \|\boldsymbol{\sigma}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}((\alpha,t,s)) - \boldsymbol{\sigma}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}((\alpha,t,s))\| = \sup_{\alpha\in I} \max \left\{ \begin{array}{l} \sup_{(t,s)\in U_\alpha\cap V_\alpha} 0, \\ \sup_{(t,s)\in U_\alpha\backslash V_\alpha} \min_{(\mathbf{v}_a,v_b)\in V_\alpha} d((\mathbf{v}_a,v_b),(t,s)), \\ \sup_{(t,s)\in V_\alpha\backslash U_\alpha} \min_{(\mathbf{u}_a,u_b)\in U_\alpha} d((\mathbf{u}_a,u_b),(t,s)), \\ \sup_{(t,s)\in (U_\alpha\cup V_\alpha)^c} 0 \end{array} \right\}. \tag{C.2}$$

By definition, $\sup_{\mathbf{t}\in T} \|\boldsymbol{\sigma}_{U;V}(\mathbf{t}) - \boldsymbol{\sigma}_{V;U}(\mathbf{t})\| = \delta^\Sigma(\boldsymbol{\sigma}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\sigma}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}})$. Since the cost function is the same for both $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ and $\boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}$, then $\sup_{\mathbf{t}\in T} \|\boldsymbol{\sigma}_{U;V}(\mathbf{t}) - \boldsymbol{\sigma}_{V;U}(\mathbf{t})\| = \delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}})$.

Note that

$$\sup_{(t,s)\in U_\alpha\backslash V_\alpha} \min_{(\mathbf{v}_a,v_b)\in V_\alpha} d((\mathbf{v}_a,v_b),(t,s)) = \sup_{(t,s)\in U_\alpha} \min_{(\mathbf{v}_a,v_b)\in V_\alpha} d((\mathbf{v}_a,v_b),(t,s)),$$

because if $(t,s) \in V_\alpha$, then $\min_{(\mathbf{v}_a,v_b)\in V_\alpha} d((\mathbf{v}_a,v_b),(t,s)) = 0$. Analogously,

$$\sup_{(t,s)\in V_\alpha\backslash U_\alpha} \min_{(\mathbf{u}_a,u_b)\in U_\alpha} d((\mathbf{u}_a,u_b),(t,s)) = \sup_{(t,s)\in V_\alpha} \min_{(\mathbf{u}_a,u_b)\in U_\alpha} d((\mathbf{u}_a,u_b),(t,s)).$$

Finally, replacing the 'min' with 'inf' in (C.2):

$$\begin{aligned} &\delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}) = \sup_{\mathbf{t}\in T} \|\boldsymbol{\sigma}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}(\mathbf{t}) - \boldsymbol{\sigma}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}(\mathbf{t})\| \\ &= \sup_{\alpha\in I} \left\{ \sup_{(t,s)\in U_\alpha} \inf_{(\mathbf{v}_a,v_b)\in V_\alpha} d((\mathbf{v}_a,v_b),(t,s)), \sup_{(t,s)\in V_\alpha} \inf_{(\mathbf{u}_a,u_b)\in U_\alpha} d((\mathbf{u}_a,u_b),(t,s)) \right\} \\ &= \sup_{\alpha\in I} d_H(U_\alpha, V_\alpha), \end{aligned}$$

as was to be shown. ∎

Appendix D. Lipschitz constant invariance.

REMARK D.1. *Note that in [6], Σ_s refers to "strongly infeasible", which conflicts with the notation of Π_s which refers to "solvable". For the remainder of this proof, we will use Σ_{si} to denote strongly infeasible to distinguish it from Π_s. The set of strongly infeasible problems refers to the systems which contain a finite subset of constraints that is infeasible. With this definition, it is clear that Σ_{si} is non-empty, because the constraint system with $\langle 0_n, \mathbf{x}\rangle \geq 1$ as a constraint is strongly infeasible. For brevity, write $H(\boldsymbol{\sigma}_0)$ as H.*

We first present two technical lemmas that we will use in our proof of Lipschitz constant invariance.

LEMMA D.2 (Distance to ill-posedness). *If $\boldsymbol{\pi} := (\mathbf{c}, \boldsymbol{\sigma}_0) \in \Pi_f$, then*

$$\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) = d(0_{n+1}, \mathrm{bd}(H(\boldsymbol{\sigma}_0))). \tag{D.1}$$

Proof. We need only show that $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) = \delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_{si}))$, because

$$\delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_{si})) = d(0_{n+1}, \text{bd}(H))$$

follows from Theorem 6 in [6].

First, we show $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) \leq \delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_{si}))$. Since $\Sigma_{si} \subseteq \Sigma_i$ and $\boldsymbol{\sigma}_0 \notin \Sigma_{si}$, then $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) \leq \delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_{si})$. By Corollary 1 in [6], since $\boldsymbol{\sigma}_0 \notin \Sigma_{si} \subsetneq \Sigma$, we have that $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_{si}) = \delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_{si}))$. Thus, $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) \leq \delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_{si}) = \delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_{si}))$. To prove the reverse inequality, first note that by Corollary 1 in [6], $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) = \delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_f))$. For all $\boldsymbol{\sigma} \in \text{bd}(\Sigma_f)$, we have $\boldsymbol{\sigma} \notin \Sigma_\infty$ by definition. Thus, by Theorem 5 part (iii) from [6], for all $\boldsymbol{\sigma} \in \text{bd}(\Sigma_f)$, we have $\boldsymbol{\sigma} \in \text{bd}(\Sigma_{si})$. This means that $\delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_f)) \geq \delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_{si}))$. ∎

REMARK D.3. *This lemma gives us a way to calculate the quantity $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i)$ in a less abstract space.*

LEMMA D.4. *For any $\boldsymbol{\pi}_0 := (\mathbf{c}^0, \boldsymbol{\sigma}_0) \in \text{int}(\Pi_s)$, we have:*
i. $d(0_{n+1}, \text{bd}(H(\boldsymbol{\sigma}_0))) > 0$,
ii. $d(0_n, \text{bd}(Z^-(\boldsymbol{\pi}_0))) > 0$,
iii. $\delta^\Pi(\boldsymbol{\pi}_0, \text{bd}(\Pi_s)) > 0$,
iv. Let any $0 < \epsilon < \delta^\Pi(\boldsymbol{\pi}_0, \text{bd}(\Pi_s))$ be given, then $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) > \epsilon$.
In particular, the values in Definition 3.3 are well-defined for $0 < \epsilon < \delta^\Pi(\boldsymbol{\pi}_0, \text{bd}(\Pi_s))$.

Proof. We prove each of the points individually:
i. The required statement is equivalent to proving $0_{n+1} \notin \text{bd}(H(\boldsymbol{\sigma}_0))$. By Lemma B.2, $\boldsymbol{\pi}_0 \notin \Pi_\infty$, since $\boldsymbol{\pi}_0 \in \text{int}(\Pi_f)$. Proposition 2(ii) from [7] states that if $\boldsymbol{\pi}_0 \in \Pi \backslash \Pi_\infty$, then $0_{n+1} \in \text{ext}(H(\boldsymbol{\sigma}_0))$ if and only if $\boldsymbol{\pi}_0 \in \text{int}(\Pi_f)$. Since $\boldsymbol{\pi}_0 \in \text{int}(\Pi_s) \subseteq \text{int}(\Pi_f)$, we immediately have $0_{n+1} \in \text{ext}(H(\boldsymbol{\sigma}_0))$.
ii. The required statement is equivalent to $0_n \notin \text{bd}(Z^-(\boldsymbol{\pi}_0))$. Proposition 3(i) from [7] gives $0_{n+1} \in \text{int}(Z^-(\boldsymbol{\pi}_0))$ if and only if $\boldsymbol{\pi}_0 \in \text{int}(\Pi_s)$.
iii. Theorem 2 from [7] states that if $\boldsymbol{\pi} \in \text{cl}(\Pi_s)$, then

$$\delta^\Pi(\boldsymbol{\pi}_0, \text{bd}(\Pi_s)) = \min\{d(0_{n+1}, \text{bd}(H(\boldsymbol{\sigma}_0))), d(0_n, \text{bd}(Z^-(\boldsymbol{\pi}_0)))\}.$$

Note that $\boldsymbol{\pi} \in \text{int}(\Pi_s) \subseteq \text{cl}(\Pi_s)$. Thus, $\delta^\Pi(\boldsymbol{\pi}_0, \text{bd}(\Pi_s)) > 0$.
iv. Let ϵ be such that $0 < \epsilon < \delta^\Pi(\boldsymbol{\pi}_0, \text{bd}(\Pi_s))$ be given. By assumption, $\boldsymbol{\pi}_0 \in \Pi_f$, so that $\boldsymbol{\sigma}_0 \in \Sigma_f$. Then, by Theorem 2 from [7], $d(0_{n+1}, \text{bd}(H(\boldsymbol{\sigma}_0))) \geq \delta^\Pi(\boldsymbol{\pi}_0, \text{bd}(\Pi_s))$, noting that $\boldsymbol{\pi}_0 \in \text{int}(\Pi_s) \subseteq \text{cl}(\Pi_s)$. Lastly, Lemma D.2 implies that $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) = d(0_{n+1}, \text{bd}(H(\boldsymbol{\sigma}_0))) \geq \delta^\Pi(\boldsymbol{\pi}_0, \text{bd}(\Pi_s)) > \epsilon$.

For the "in particular" statement, we need only check that the denominators are non-zero and that $\nu(\boldsymbol{\pi}_0)$ is finite. Since we are using the Euclidean norm,

$$d_*(0_n, \text{bd}(Z^-(\boldsymbol{\pi}_0))) = d(0_n, \text{bd}(Z^-(\boldsymbol{\pi}_0)))$$

Now, $d(0_n, \text{bd}(Z^-(\boldsymbol{\pi}_0))) \geq \delta^\Pi(\boldsymbol{\pi}_0, \text{bd}(\Pi_s)) > \epsilon > 0$ due to Theorem 2 from [7]. Also, $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) > \epsilon > 0$ due to the fourth statement. Lastly, $\boldsymbol{\pi}_0 \in \text{int}(\Pi_s)$ implies that $\nu(\boldsymbol{\pi}_0)$ is finite. ∎

Now we proceed to prove that given any two LSIO problems $\boldsymbol{\pi}_1, \boldsymbol{\pi}_2 \in \text{int}(\Pi_s)$, such that $\boldsymbol{\pi}_1 \sim_\Pi \boldsymbol{\pi}_2$, then the Lipschitz constants in Theorem 3.4 (Theorem 4.3 from [9]) satisfy $L(\boldsymbol{\pi}_1, \epsilon) = L(\boldsymbol{\pi}_2, \epsilon)$.

LEMMA D.5. *Let $\boldsymbol{\pi}_1 = (\mathbf{c}^1, \boldsymbol{\sigma}_1) \in \text{int}(\Pi_s)$ and $\boldsymbol{\pi}_2 = (\mathbf{c}^2, \boldsymbol{\sigma}_2) \in \text{int}(\Pi_s)$. Suppose $\boldsymbol{\pi}_1 \sim_\Pi \boldsymbol{\pi}_2$. Then:*

i. $\delta^{\Pi}(\boldsymbol{\pi}_1, \mathrm{bd}(\Pi_s)) = \delta^{\Pi}(\boldsymbol{\pi}_2, \mathrm{bd}(\Pi_s))$.

ii. $\delta^{\Sigma}(\boldsymbol{\sigma}_1, \Sigma_i) = \delta^{\Sigma}(\boldsymbol{\sigma}_2, \Sigma_i)$ *and furthermore, if* $0 < \epsilon < \delta^{\Pi}(\boldsymbol{\pi}_1, \mathrm{bd}(\Pi_s)) = \delta^{\Pi}(\boldsymbol{\pi}_2, \mathrm{bd}(\Pi_s))$, *then* $L(\boldsymbol{\pi}_1, \epsilon) = L(\boldsymbol{\pi}_2, \epsilon)$.

Proof. i. By the definition of $\boldsymbol{\pi}_1 \sim_{\Pi} \boldsymbol{\pi}_2$, we have $\mathbf{c}^1 = \mathbf{c}^2$, and $\boldsymbol{\sigma}_1 := (\mathbf{a}_t^1, b_t^1)_{t\in T} \sim_{\Sigma} \boldsymbol{\sigma}_2 := (\mathbf{a}_t^2, b_t^2)_{t\in T}$, that is, $\{(\mathbf{a}_t^1, b_t^1), t \in T\} = \{(\mathbf{a}_t^2, b_t^2), t \in T\}$. By the definition of $H(\cdot)$, it is clear that $H(\boldsymbol{\sigma}_1) = H(\boldsymbol{\sigma}_2)$, which implies that $d(0_{n+1}, \mathrm{bd}(H(\boldsymbol{\sigma}_1))) = d(0_{n+1}, \mathrm{bd}(H(\boldsymbol{\sigma}_2)))$. Similarly, $Z^-(\boldsymbol{\pi}_1) = Z^-(\boldsymbol{\pi}_2)$, which implies that $d(0_n, \mathrm{bd}(Z^-(\boldsymbol{\pi}_1))) = d(0_n, \mathrm{bd}(Z^-(\boldsymbol{\pi}_2)))$. Since both $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2 \in \mathrm{int}(\Pi_s) \subseteq \mathrm{cl}(\Pi_s)$, we can apply Theorem 2 from [7]. This theorem states that, for $\boldsymbol{\pi} := (\mathbf{c}, \boldsymbol{\sigma}) \in \mathrm{cl}(\Pi_s)$,

$$\delta^{\Pi}(\boldsymbol{\pi}, \mathrm{bd}(\Pi_s)) = \min\left\{d(0_{n+1}, \mathrm{bd}(H(\boldsymbol{\sigma}))), d(0_n, \mathrm{bd}(Z^-(\boldsymbol{\pi})))\right\},$$

which implies that $\delta^{\Pi}(\boldsymbol{\pi}_1, \mathrm{bd}(\Pi_s)) = \delta^{\Pi}(\boldsymbol{\pi}_2, \mathrm{bd}(\Pi_s))$.

ii. As shown in the proof of part 1 of this Lemma,

$$d(0_{n+1}, \mathrm{bd}(H(\boldsymbol{\sigma}_1))) = d(0_{n+1}, \mathrm{bd}(H(\boldsymbol{\sigma}_2))),$$

so by Lemma D.2, $\delta^{\Sigma}(\boldsymbol{\sigma}_1, \Sigma_i) = \delta^{\Sigma}(\boldsymbol{\sigma}_2, \Sigma_i)$. Finally, $L(\boldsymbol{\pi}_1, \epsilon) = L(\boldsymbol{\pi}_2, \epsilon)$ for Π-equivalent problems $\boldsymbol{\pi}_1, \boldsymbol{\pi}_2$ follows from the definition of $L(\cdot, \epsilon)$ and all the supporting quantities in Definition 3.2 and Definition 3.3. ∎

Appendix E. Proofs for Section 5.

E.1. Proof of Theorem 5.3.

Proof. The proof is nearly identical to that of Theorem 5.1. The only differences are that T is defined as $I \times \mathbb{R}^n \times \mathbb{R}$ and Theorem 4.4 is used in place of Theorem 4.2 where appropriate. The full proof is included here for completeness.

Let $T := I \times \mathbb{R}^n \times \mathbb{R}$. Denote the elements $\mathbf{t} \in T$ as tuples $\mathbf{t} := (\alpha, t, s)$ where $\alpha \in I$, $t \in \mathbb{R}^n$ and $s \in \mathbb{R}$.

1. *Write* $\mathbf{RO}(\widehat{\mathbf{U}})$ *as the LSIO problem* $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$. First, we show that $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ and $\mathbf{RO}(\widehat{\mathbf{U}})$ are equivalent. Note that the cost functions for the two problems are the same, and that $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ contains all the constraints of $\mathbf{RO}(\widehat{\mathbf{U}})$ plus some additional trivial constraints. Thus $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ and $\mathbf{RO}(\widehat{\mathbf{U}})$ have the same feasible solution set, optimal value, and optimal solution sets. Furthermore, since $\mathbf{RO}(\widehat{\mathbf{U}})$ satisfies the strong Slater condition with Slater constant ρ, and since $\langle 0_n, \mathbf{x}\rangle \geq -\rho + \rho = 0$ for all $\mathbf{x}$, it follows that $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ satisfies the strong Slater condition with Slater constant ρ.
   Second, we show that $\boldsymbol{\pi}_{\widehat{\mathbf{U}}} \in \mathrm{int}(\Pi_s)$. By assumption, $F^{\mathrm{opt}}(\mathbf{RO}(\widehat{\mathbf{U}}))$ is non-empty and bounded, so $F^{\mathrm{opt}}(\boldsymbol{\pi}_{\widehat{\mathbf{U}}})$ is non-empty and bounded. Since $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ satisfies the strong Slater condition, by Theorem 3.5 (Theorem 3.1 (i),(ii), and (vi) from [16]), we have $\boldsymbol{\pi}_{\widehat{\mathbf{U}}} \in \mathrm{int}(\Pi_f)$. Thus, by Proposition 1 part (vi) from [8], $\mathbf{c} \in \mathrm{int}(\mathrm{cone}(\{a_{\mathbf{t}} : \mathbf{t} \in T\}))$, and by part (vii), this implies that $\boldsymbol{\pi}_{\widehat{\mathbf{U}}} \in \mathrm{int}(\Pi_s)$.
2. *Choose* $\widehat{\mathbf{V}}$ *in an* ϵ*-neighborhood of* $\widehat{\mathbf{U}}$. Let $\epsilon > 0$ be given satisfying $0 < \epsilon < \delta^{\Pi}(\boldsymbol{\pi}_{\widehat{\mathbf{U}}}, \mathrm{bd}(\Pi_s))$. Such an ϵ exists because

$$\delta^{\Pi}(\boldsymbol{\pi}_{\widehat{\mathbf{U}}}, \mathrm{bd}(\Pi_s)) > 0 \tag{E.1}$$

   by Lemma D.4. Let $\widehat{\mathbf{V}} = \Pi_{\alpha\in I} V_\alpha$ be given with non-empty, compact, and convex $V_\alpha \subseteq \mathbb{R}^{n+1}$ satisfying $d_\natural(\widehat{\mathbf{U}}, \widehat{\mathbf{V}}) \le \epsilon < \delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}}}, \mathrm{bd}(\Pi_s))$. Such a $\widehat{\mathbf{V}}$ exists; e.g., the choice $V_\alpha := U_\alpha + B(0, \epsilon/2)$ for all $\alpha \in I$ satisfies $d_\natural(\widehat{\mathbf{U}}, \widehat{\mathbf{V}}) < \epsilon$.

3. *Define* $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}} := (\mathbf{c}, \boldsymbol{\sigma}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}})$. Define the LSIO problem $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}} := (\mathbf{c}, \boldsymbol{\sigma}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}})$ as in Theorem 4.4. By the same theorem, $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ is well-defined. Every constraint in $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ is in $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ and vice versa because for each $\alpha \in I$, $(t,s) \in V_\alpha \backslash U_\alpha$, we have that $\mathrm{argmin}_{(\mathbf{u}_a, u_b) \in U_\alpha} d((\mathbf{u}_a, u_b), (t,s)) \in U_\alpha$. Thus, $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}} \sim_\Pi \boldsymbol{\pi}_{\widehat{\mathbf{U}}}$. Furthermore, since $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}} \sim_\Pi \boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ and $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ satisfies the strong Slater condition with the Slater constant ρ, it follows that $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ also satisfies the strong Slater condition with constant ρ.
4. *Define* $\boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}} := (\mathbf{c}, \boldsymbol{\sigma}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}})$. Define the LSIO problem $\boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}} := (\mathbf{c}, \boldsymbol{\sigma}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}})$ as in Theorem 4.4. By the same theorem, $\boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}$ is well-defined and is equivalent to $\mathbf{RO}(\widehat{\mathbf{V}})$.
5. *By Theorem 4.4,* $\delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}) = \mathbf{d}_\natural(\widehat{\mathbf{U}}, \widehat{\mathbf{V}}) < \epsilon.$
6. *Apply Theorem 3.4 (Theorem 4.3 from [9]) with* $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}} \mapsto \boldsymbol{\pi}_0, \boldsymbol{\pi}_1$ *and* $\boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}} \mapsto \boldsymbol{\pi}_2$. We check that the assumptions of Theorem 3.4 are satisfied:
   (a) $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}} \in \mathrm{int}(\Pi_s)$. This proof is identical to the proof that $\boldsymbol{\pi}_{\widehat{\mathbf{U}}} \in \mathrm{int}(\Pi_s)$, given in Step 1.
   (b) $\epsilon < \delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \mathrm{bd}(\Pi_s))$. Recall that $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$ and $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ are both in $\mathrm{int}(\Pi_s)$, have non-empty bounded optimal solution sets and are Π-equivalent to each other. Thus, we have $\epsilon < \delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}}}, \mathrm{bd}(\Pi_s)) = \delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \mathrm{bd}(\Pi_s))$, where the inequality is by assumption and the equality is by Lemma D.5.
   (c) $\delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}) \le \epsilon$ *and* $\delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}) = 0 \le \epsilon$. The first inequality comes from Step 5, and the second inequality is trivial.

   Thus, the assumptions of Theorem 3.4 are satisfied, so:
$$|\nu(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}) - \nu(\boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}})| \le L(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \epsilon)\delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}),$$
   where $L(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \epsilon)$, as defined in Definition 3.3.
7. *Lastly, show that the Lipschitz constant is independent of the choice of* V. Applying Lemma D.5 to Π-equivalent problems $\boldsymbol{\pi}_{\widehat{\mathbf{U}}}$ and $\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}$, it follows that $L(\boldsymbol{\pi}_{\widehat{\mathbf{U}}}, \epsilon) = L(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \epsilon)$. Now define $L(\widehat{\mathbf{U}}, \epsilon) := L(\boldsymbol{\pi}_{\widehat{\mathbf{U}}}, \epsilon)$. By recognizing that
$$\nu(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}) = \nu(\mathbf{RO}(\widehat{\mathbf{U}})) \text{ and } \nu(\boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}) = \nu(\mathbf{RO}(\widehat{\mathbf{V}})),$$
   and using $\delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}) = d_H(\widehat{\mathbf{U}}, \widehat{\mathbf{V}})$, we obtain the final result:
$$\begin{aligned}
|\nu(\mathbf{RO}(\widehat{\mathbf{U}})) - \nu(\mathbf{RO}(\widehat{\mathbf{V}}))| &= |\nu(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}) - \nu(\boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}})|, \\
&\quad \text{(By definition of } \boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}} \text{ and } \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}) \\
&\le L(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \epsilon)\delta^\Pi(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \boldsymbol{\pi}_{\widehat{\mathbf{V}};\widehat{\mathbf{U}}}), \\
&\quad \text{(By Theorem 3.4)} \\
&= L(\boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}}, \epsilon) d_H(\widehat{\mathbf{U}}, \widehat{\mathbf{V}}), \\
&\quad \text{(By Theorem 4.4)} \\
&= L(\widehat{\mathbf{U}}, \epsilon) d_H(\widehat{\mathbf{U}}, \widehat{\mathbf{V}}). \\
&\quad \text{(Since } \boldsymbol{\pi}_{\widehat{\mathbf{U}};\widehat{\mathbf{V}}} \sim_\Pi \boldsymbol{\pi}_{\widehat{\mathbf{U}}})
\end{aligned}$$

The proof is complete. ∎

Appendix F. Proofs for Section 7.

F.1. Lipschitz continuity of the ϵ-optimal solution set for LSIO problems. Before we show Lipschitz continuity of the ϵ-optimal solution set for RO problems, we first establish an analogous Lipschitz continuity result for LSIO problems, which is the focus of this subsection. Next, we present Lemma F.1 and Lemma F.2, which combine results from [23] and apply them to LSIO problems. Lemma F.1 relates the distance between ϵ-optimal solution sets with the distance between feasible solution sets and Lemma F.2 relates the distance between feasible solution sets and the δ^{Π}-distance between the LSIO problems. Combining Lemma F.1 and Lemma F.2 gives us the required Lipschitz continuity of the ϵ-optimal solution set for LSIO problems.

LEMMA F.1 (Dist. b/w ϵ-optimal solution sets via $F(\cdot)$). *Suppose $\boldsymbol{\pi}_1 := (\mathbf{c}, \boldsymbol{\sigma}_1)$ and $\boldsymbol{\pi}_2 := (\mathbf{c}, \boldsymbol{\sigma}_2)$ are LSIO problems such that there exists an $r_0 > 0$ such that $r_0 B \cap F^{\text{opt}}(\boldsymbol{\pi}_1) \neq \varnothing$ and $r_0 B \cap F^{\text{opt}}(\boldsymbol{\pi}_2) \neq \varnothing$ and $\nu(\boldsymbol{\pi}_1) > -r_0$ and $\nu(\boldsymbol{\pi}_2) > -r_0$. Define the extended real-valued functions $f_1(\mathbf{x}) := \langle \mathbf{c}, \mathbf{x} \rangle + \chi_{F(\boldsymbol{\sigma}_1)}(\mathbf{x})$ and $f_2(\mathbf{x}) := \langle \mathbf{c}, \mathbf{x} \rangle + \chi_{F(\boldsymbol{\sigma}_2)}(\mathbf{x})$, where χ is the characteristic function. Then, for all $r > r_0$ and for all $\epsilon > 0$:*

$$\widehat{\mathbf{d}}_r(\epsilon\text{-argmin} f_1, \epsilon\text{-argmin} f_2) \leq (1 + 4r\epsilon^{-1})(1 + \|\mathbf{c}\|)\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2)).$$

Proof. The outline of the proof is as follows. We first construct unconstrained convex functions f_1 and f_2 whose ϵ-minimizers coincide with ϵ-optimal solutions of $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$. Next, we use Theorem 3.7 (Theorem 7.69 from [23]) to obtain an inequality relating $\widehat{\mathbf{d}}_r(\epsilon\text{-argmin} f_1, \epsilon\text{-argmin} f_2)$ to the epigraph distance $\widehat{\delta}_r^+(f_1, f_2)$. Finally, we use Example 7.62 and Proposition 4.37 from [23] to further bound $\widehat{\delta}_r^+(f_1, f_2)$ by $\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2))$.

1. *Define $f_1(\mathbf{x}) := \langle \mathbf{c}, \mathbf{x} \rangle + \chi_{F(\boldsymbol{\sigma}_1)}(\mathbf{x})$ and $f_2(\mathbf{x}) := \langle \mathbf{c}, \mathbf{x} \rangle + \chi_{F(\boldsymbol{\sigma}_2)}(\mathbf{x})$.* We will show that f_1 and f_2 are proper, lower semicontinuous, and convex functions and that the minimizers (and ϵ-minimizers) of f_1 and f_2 coincide with the optimal (and ϵ-optimal) solutions of $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$. Since $F(\boldsymbol{\sigma}_1)$ and $F(\boldsymbol{\sigma}_2)$ are non-empty, there exist $\mathbf{x}$ and $\mathbf{x}'$ such that $f_1(\mathbf{x}) < +\infty$ and $f_2(\mathbf{x}') < +\infty$. Thus, f_1 and f_2 are proper functions. By Theorem 1.6 from [23], f_1 and f_2 are lower semicontinuous if and only if $\text{lev}_{\leq \alpha} f_1$ and $\text{lev}_{\leq \alpha} f_2$ are closed for all $\alpha \in \mathbb{R}$, where

$$\text{lev}_{\leq \alpha} f_i := \{\mathbf{x} \in \mathbb{R}^n : f_i(\mathbf{x}) \leq \alpha\} = \{\mathbf{x} \in F(\boldsymbol{\sigma}_i) : \langle \mathbf{c}, \mathbf{x} \rangle \leq \alpha\}, \text{ for } i = 1, 2.$$

   Let $f_{01}(\mathbf{x}) := \langle \mathbf{c}, \mathbf{x} \rangle$. Since f_{01} is continuous, $f_{01}^{-1}((-\infty, \alpha]) =: \text{lev}_{\leq \alpha} f_{01}$ is closed. Since $F(\boldsymbol{\sigma}_1)$ is closed, $\text{lev}_{\leq \alpha} f_1 = F(\boldsymbol{\sigma}_1) \cap \text{lev}_{\leq \alpha} f_{01}$ is closed. The proof for $\text{lev}_{\leq \alpha} f_2$ is identical. Note that the characteristic function χ_A is convex if A is convex. Thus, f_1 and f_2 are convex because they are the sum of convex functions. Finally, by construction, the minimizers (and ϵ-minimizers) of f_1 and f_2 are exactly the optimal solutions (and ϵ-optimal solutions) of $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$, because $f_1(\mathbf{x}) = +\infty$ whenever $\mathbf{x} \notin F(\boldsymbol{\sigma}_1)$ and $f_2(\mathbf{x}) = +\infty$ whenever $\mathbf{x} \notin F(\boldsymbol{\sigma}_2)$. Thus, minimizing f_1 and f_2 is the same as minimizing $\langle \mathbf{c}, \mathbf{x} \rangle$ over $F(\boldsymbol{\sigma}_1)$ and $F(\boldsymbol{\sigma}_2)$, respectively.
2. *Use Theorem 3.7 (Theorem 7.69 from [23]) to get, for all $r > r_0$,*

$$\widehat{\mathbf{d}}_r(\epsilon\text{-argmin} f_1, \epsilon\text{-argmin} f_2) \leq (1 + 4r\epsilon^{-1})\widehat{\delta}_r^+(f_1, f_2). \tag{F.1}$$

   We apply Theorem 3.7 (Theorem 7.69 from [23]) to f_1 and f_2. Theorem 3.7 requires that f_1 and f_2 are proper, lower semicontinuous, and convex functions, which was shown earlier. The other conditions in Theorem 3.7 are satisfied by our assumptions, and the fact that the minimizers of f_1 and f_2 coincide with the optimal solutions of $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$, so that f_1 and f_2 satisfy $\text{argmin} f_1 \cap r_0 B \neq \varnothing$ and $\text{argmin} f_2 \cap r_0 B \neq \varnothing$ and $\min f_1 \geq -r_0$ and $\min f_2 \geq -r_0$.

3. *Use Example 7.62 from [23] to get:* $\widehat{\delta}_r^+(f_1, f_2) \leq (1 + \|\mathbf{c}\|)\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2))$. We apply the results of Example 7.62 from [23] to f_1 and f_2, by setting $\langle \mathbf{c}, \mathbf{x}\rangle \mapsto f_{01} = f_{02}$, $F(\boldsymbol{\sigma}_1) \mapsto C_1$ and $F(\boldsymbol{\sigma}_2) \mapsto C_2$. For any $r > 0$ the constant $\kappa_i(r)$ from Example 7.62 can be chosen uniformly to be $\|\mathbf{c}\|$, since $f_{01} = f_{02} = \langle \mathbf{c}, \mathbf{x}\rangle$ so that $|\langle \mathbf{c}, \overline{\mathbf{x}}\rangle - \langle \mathbf{c}, \mathbf{x}\rangle| \leq \|\mathbf{c}\|\|\overline{\mathbf{x}} - \mathbf{x}\|$ by the Cauchy-Schwarz inequality. Furthermore, $f_{01} = f_{02}$ so that $|f_{01}(\mathbf{x}) - f_{02}(\mathbf{x})| = 0$ for all $\mathbf{x}$. Thus, for all $0 < r < r' < +\infty$ and $\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2)) < r' - r$:

$$\widehat{\delta}_r^+(f_1, f_2) \leq (1 + \|\mathbf{c}\|)\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2)).$$

If we apply Proposition 4.37 parts (a) and (c) from [23], we obtain

$$\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2)) \leq \mathbf{d}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2))) \leq \max\{d(0, F(\boldsymbol{\sigma}_1)), d(0, F(\boldsymbol{\sigma}_2))\} + r.$$

Thus, for *any* given $r > 0$, by choosing $r' := \max\{d(0, F(\boldsymbol{\sigma}_1)), d(0, F(\boldsymbol{\sigma}_2))\} + 2r$, we have the following inequality, independent of r':

$$\widehat{\delta}_r^+(f_1, f_2) \leq (1 + \|\mathbf{c}\|)\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2)). \tag{F.2}$$

4. *Combine the inequalities.* Using the inequalities (F.1) and (F.2) it follows that, for all $r > r_0$:

$$\widehat{\mathbf{d}}_r(\epsilon\text{-argmin} f_1, \epsilon\text{-argmin} f_2) \leq (1 + 4r\epsilon^{-1})(1 + \|\mathbf{c}\|)\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2)).$$

The proof is complete. ∎

LEMMA F.2 (Dist. b/w feasible solution sets). *Let* $\boldsymbol{\pi}_0 := (\mathbf{c}, \boldsymbol{\sigma}_0) \in \text{int}(\Pi_f)$ *be given. Let* η *satisfying* $0 < \eta < \delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i)$ *be given. For all* $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2 \in \Sigma_f$ *satisfying* $\delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0) \leq \eta$ *and* $\delta^\Sigma(\boldsymbol{\sigma}_2, \boldsymbol{\sigma}_0) \leq \eta$, *the following inequality holds for* $\boldsymbol{\pi}_1 := (\mathbf{c}, \boldsymbol{\sigma}_1)$ *and* $\boldsymbol{\pi}_2 := (\mathbf{c}, \boldsymbol{\sigma}_2)$ *and for all* $r > 0$*:*

$$\begin{aligned}\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2)) &\leq \left(\sup_{\mathbf{z} \in rB} \frac{\psi(\|\mathbf{z}\|)}{\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) - \eta}\right) \delta^\Pi(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2),\\ &= \left(\frac{(1 + r)\sqrt{1 + r^2}}{\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) - \eta}\right) \delta^\Pi(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2).\end{aligned}$$

Proof. For brevity, write $F(\boldsymbol{\sigma}_1)$ and $F(\boldsymbol{\sigma}_2)$ as F_1 and F_2, respectively. By (7.1b),

$$\widehat{\mathbf{d}}_r(F_1, F_2) := \max\{e((F_1)_r, F_2), e((F_2)_r, F_1)\}.$$

From observing the definition of $\widehat{\mathbf{d}}_r$, we need only prove the desired inequality for $e((F_1)_r, F_2)$, and omit the analogous proof for $e((F_2)_r, F_1)$. Let η satisfying $0 < \eta < \delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i)$ and $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2 \in \Sigma_f$ satisfying $\delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0) \leq \eta$ and $\delta^\Sigma(\boldsymbol{\sigma}_2, \boldsymbol{\sigma}_0) \leq \eta$ be given.

1. *First,* $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2 \in \text{int}(\Sigma_f)$. By assumption, $\boldsymbol{\sigma}_0 \in \text{int}(\Sigma_f)$. By Corollary 1 from [6], since $\boldsymbol{\sigma}_0 \in \Sigma_f$, we have $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) = \delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_f))$. Suppose to the contrary that, $\boldsymbol{\sigma}_1 \in \text{bd}(\Sigma_f)$. Then by definition $\delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_f)) \leq \delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_0) \leq \eta < \delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) = \delta^\Sigma(\boldsymbol{\sigma}_0, \text{bd}(\Sigma_f))$, which is a contradiction. The same reasoning holds for $\boldsymbol{\sigma}_2$.

2. *Then,* $\sup_{\mathbf{z}^1 \in F_1 \cap rB} d(\mathbf{z}^1, F_2) \leq \sup_{\mathbf{z}^1 \in F_1 \cap rB} \frac{\psi(\|\mathbf{z}^1\|)}{\delta^\Sigma(\boldsymbol{\sigma}_2, \Sigma_i)} \delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)$.
   Apply Corollary 4.1 from [9] with $\boldsymbol{\sigma}_1 \mapsto \boldsymbol{\sigma}_0$ ($\boldsymbol{\sigma}_0$ in this instance referring to the $\boldsymbol{\sigma}_0$ in *that* Corollary) and $\boldsymbol{\sigma}_2 \mapsto \boldsymbol{\sigma}$, with the required assumption that $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2 \in \text{int}(\Sigma_f)$ being satisfied. Thus, for any $\mathbf{z}^1 \in F_1$, we have

$$d(\mathbf{z}^1, F_2) \leq \frac{\psi(\|\mathbf{z}^1\|)}{\delta^\Sigma(\boldsymbol{\sigma}_2, \Sigma_i)} \delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2).$$

   In particular, this inequality holds for any $\mathbf{z}^1 \in F_1 \cap rB$:

$$\sup_{\mathbf{z}^1 \in F_1 \cap rB} d(\mathbf{z}^1, F_2) \leq \sup_{\mathbf{z}^1 \in F_1 \cap rB} \frac{\psi(\|\mathbf{z}^1\|)}{\delta^\Sigma(\boldsymbol{\sigma}_2, \Sigma_i)} \delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2). \tag{F.3}$$

3. *Then* $e((F_1)_r, F_2) \leq \sup_{\mathbf{z} \in rB} \frac{\psi(\|\mathbf{z}\|)}{\delta^\Sigma(\boldsymbol{\sigma}_2, \Sigma_i)} \delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)$. The expression in the left hand side of (F.3) is precisely the definition of $e((F_1)_r, F_2)$. The right hand side of (F.3) is at most the supremum over $\mathbf{z} \in rB$:

$$e((F_1)_r, F_2) \leq \sup_{\mathbf{z} \in rB} \frac{\psi(\|\mathbf{z}\|)}{\delta^\Sigma(\boldsymbol{\sigma}_2, \Sigma_i)} \delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2).$$

4. *Finally,* $e((F_1)_r, F_2) \leq \sup_{\mathbf{z} \in rB} \frac{\psi(\|\mathbf{z}\|)}{\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) - \eta} \delta^\Pi(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2)$. We first show that $\delta^\Sigma(\boldsymbol{\sigma}_2, \Sigma_i) \geq \delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) - \eta$:

$$\delta^\Sigma(\boldsymbol{\sigma}_2, \Sigma_i) := \inf_{\boldsymbol{\sigma}' \in \Sigma_i} \delta^\Sigma(\boldsymbol{\sigma}_2, \boldsymbol{\sigma}') = \inf_{\boldsymbol{\sigma}' \in \Sigma_i} \delta^\Sigma(\boldsymbol{\sigma}', \boldsymbol{\sigma}_2), \tag{F.4}$$

$$\geq \inf_{\boldsymbol{\sigma}' \in \Sigma_i} (\delta^\Sigma(\boldsymbol{\sigma}', \boldsymbol{\sigma}_0) - \delta^\Sigma(\boldsymbol{\sigma}_0, \boldsymbol{\sigma}_2)), \tag{F.5}$$

$$\geq \delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) - \eta, \tag{F.6}$$

   where (F.5) follows from the triangle inequality and (F.6) follows from the definition of $\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i)$ and $\delta^\Sigma(\boldsymbol{\sigma}_0, \boldsymbol{\sigma}_2) \leq \eta$. Since $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$ have the same cost function, $\delta^\Sigma(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2) = \delta^\Pi(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2)$.
5. *Thus, the desired inequalities hold.*
   The proof of $e((F_2)_r, F_1) \leq \sup_{\mathbf{z} \in rB} \frac{\psi(\|\mathbf{z}\|)}{\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) - \eta} \delta^\Pi(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2)$ follows by interchanging the roles of $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ in the previous steps. Thus:

$$\widehat{\mathbf{d}}_r(F(\boldsymbol{\sigma}_1), F(\boldsymbol{\sigma}_2)) \leq \left(\sup_{\mathbf{z} \in rB} \frac{\psi(\|\mathbf{z}\|)}{\delta^\Sigma(\boldsymbol{\sigma}_0, \Sigma_i) - \eta} \right) \delta^\Pi(\boldsymbol{\pi}_1, \boldsymbol{\pi}_2). \tag{F.7}$$

   The final equality follows by the definition and monotonicity of $\psi(\|\mathbf{z}\|)$ and the fact that $\|\mathbf{z}\| \leq r$ for $z \in rB$. ∎

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